Hidden Symmetries, Rapid Turns and Cosmic Acceleration

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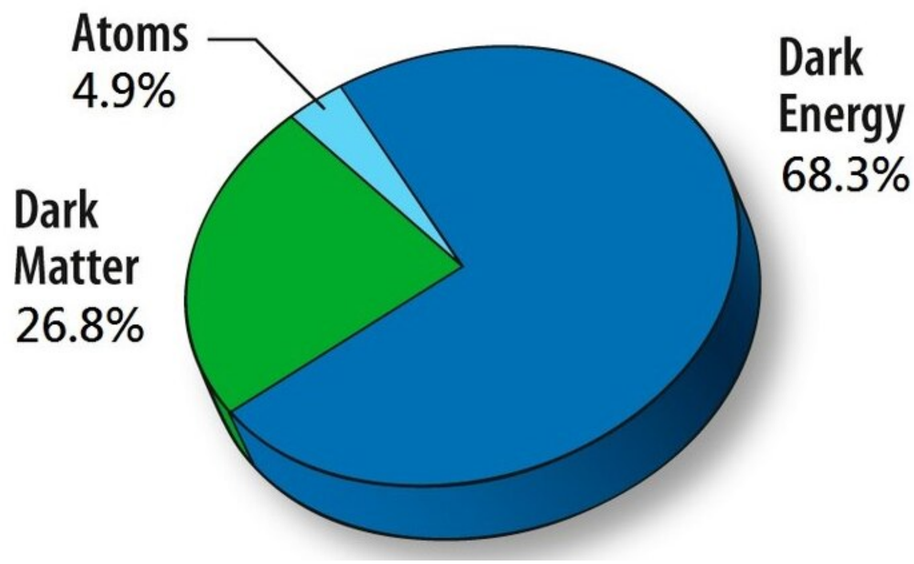
arXiv:2012.03705 [hep-th], arXiv:2111.12136 [hep-th]

(with J. Dumancic, R. Gass, L.C.R. Wijewardhana)

Motivation

Composition of the Universe today:

(from CMB observations and other sources)



Nature of Dark Energy and Dark Matter?:

Background solutions in multifield cosmology, whose field-space trajectories exhibit rapid turns

Primordial Black Holes (PBHs):

- Contribute to Dark Matter:

  Sufficient abundance of PBHs: natural candidate for DM

  (Depending on the model: from a fraction to all of DM...)

- Formed in Early Universe:

  Large enough fluctuations during cosmological inflation can seed PBHs

- Growing observational evidence for PBHs from BH mergers:

  LIGO/Virgo Collaboration: (first detection: LIGO in 2015)

Theoretical understanding of PBH-generation:

- Single-field inflationary models:

  More conventional, but PBH-formation is a challenge...

- Multi-field cosmological inflation:

  – Motivated by quantum gravity

  (string compact.: even number of scalars ; swampland conjectures...)

  – Leads to new phenomena

Goal:

PBH-generation from certain class of two-field models

(due to solutions of the EoM, which exhibit sharp turns in field space)

Two-field Cosmological Models

Action:

$$S = \int d^4x \sqrt{-\det g}\left[\frac{R}{2} - \frac{1}{2} G_{IJ}(\phi)\, g^{\mu\nu}\, \partial_\mu \phi^I\, \partial_\nu \phi^J - V(\phi)\right] ,$$

$g_{\mu\nu}(x)$ - spacetime metric , $\mu, \nu = 0, ..., 3$

$G_{IJ}(\phi)$ - field space metric , $I, J = 1, 2$

Standard background Ansatze:

$$ds_g^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad , \quad \phi^I = \phi_0^I(t) \quad ,$$

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$ - Hubble parameter

Conceptual note:

In single-field models potential $V(\phi)$ plays key role:

Always: field redefinition $\rightarrow$ canonical kinetic term

(Can transfer complexity to the potential)

In multi-field models:

Cannot redefine away the curvature of G_{IJ} !

(I.e., kinetic term becomes important !)

$\Rightarrow$ Can have: - Genuine two (or multi-) field trajectories even when $\partial_{\phi^I} V = 0$ for some I

- New phenomena due to non-geodesic motion in field space

Exact solutions with hidden symmetry

Exact solutions in multifield case: difficult to find...

L.A., E.M.Babalic, C.Lazaroiu, arXiv:1809.10563 [hep-th] :

Obtained many by using Noether symmetry method

Imposing Noether symmetry is a powerful technical tool:

- can restrict:
  - form of potential V (expected)
  - value of Gaussian curvature K_G (unexpected!)

  (hence: may help for embedding in fundamental theory)
- can lead to simplified EoMs and thus facilitate finding exact solutions (as opposed to numerical ones)

Noether symmetry:

Substituting ansatz $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $\phi^I = \phi_0^I(t)$:

$$\mathcal{L} = -3a\dot{a}^2 + a^3 \left[\frac{1}{2} G_{IJ} \dot{\phi}_0^I \dot{\phi}_0^J - V(\phi_0) \right]$$

$\rightarrow$ classical mechanical action for $\{a, \phi_0^I\}$ ds.o.f.

(Euler-L. eqs of $\mathcal{L} \equiv$ original EoMs, when imposing Hamiltonian constraint)

Consider transformation:

$q^{\hat{I}} \rightarrow Q^{\hat{I}}(q)$, where $q^{\hat{I}} \equiv \{a, \phi_0^I\}$ - gen. coord. on $\mathcal{M}$

$\rightarrow$ induces transf. on tangent bundle $T\mathcal{M}$, gen. by vec. X

(with coord. $\{q^{\hat{I}}, \dot{q}^{\hat{I}}\}$)

Symmetry condition: $\quad L_X(\mathcal{L}) = 0$

Hidden symmetry: (arXiv:1809.10563 [hep-th], arXiv:1905.01611 [hep-th])

$L_X(\mathcal{L}) = 0 \quad \Rightarrow \quad$ coupled system of PDEs

$\rightarrow$ determines $X(a, \phi_0)$ and $V(\phi_0)$

- visible symmetry: acts only on $\{\phi_0^I\}$
- hidden symmetry: acts also on a (mixes a and $\{\phi_0^I\}$!)

Consider rot.-invariant metric G_{IJ} on $\mathcal{M}_0$ (with coord. $\{\phi_0^I\}$) :

$$ds_G^2 = d\varphi^2 + f(\varphi) d\theta^2 \qquad \text{(recall: } I, J = 1, 2\text{)}$$

Showed: hidden symmetry requires hyperbolic $\mathcal{M}_0$!

Found: many classes of exact solutions for hyperbolic $\mathcal{M}_0$

Class of exact solutions: (arXiv:1809.10563 [hep-th])

Take G_{IJ} - metric on Poincaré disk $\&$ impose hidden symmetry :

$$\Rightarrow \quad f(\varphi) = \frac{1}{q^2}\sinh^2(q\,\varphi) \;\; , \;\; V(\varphi,\theta) = V_0 \cosh^2(q\,\varphi) \;\; ,$$

$$q = \sqrt{\frac{3}{8}} \quad , \quad V_0 > 0$$

Poincaré disk metric:

(α-attractor notation)

$$ds_D^2 = 6\alpha \frac{dz d\bar{z}}{(1 - z\bar{z})^2} \quad ,$$

$$z = \rho e^{i\theta} \;\; , \;\; \rho \in [0, 1) \;\; ,$$

α - arbitrary parameter ; hid. sym.: $\alpha = \frac{16}{9}$

$$\rho = \tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right) \quad \Rightarrow \quad ds_D^2 = d\varphi^2 + f(\varphi) d\theta^2$$

Primordial Black Holes

PBH generation: induced by large perturbations during inflation

Characteristics of an inflationary trajectory:

Background trajectory $(\phi_0^1(t), \phi_0^2(t))$ in field space:

- Tangent and normal vectors: T^I, N_J $\quad (\, I, J = 1, 2\,)$
- Turning rate: $\Omega = -N_I D_t T^I \quad (\, D_t T^I \equiv \dot{\phi}_0^J \, \nabla_J T^I\,)$
- Slow-roll parameters: $\varepsilon = -\frac{\dot{H}}{H^2}$, $\eta_{||} = -\frac{\ddot{H}}{2H\dot{H}}$, $\eta_\perp = \frac{\Omega}{H}$

Power spectrum of curvature perturbation:

$$\mathcal{P}_\zeta \sim \mathcal{P}_0 \, e^{c\,|\eta_\perp|} \quad , \quad c = const > 0$$

For PBH generation, need δt with: $\mathcal{P}_\zeta / \mathcal{P}_0 \sim 10^7$

(Slow roll: $\varepsilon \, , |\eta_{||}| \ll 1$; Our interest: $\eta_\perp^2 \gg 1$)

PBHs from exact solutions with hidden symmetry:

(L.A., arXiv:2012.03705 [hep-th])

Consider class of solutions on Poincaré disk (mentioned earlier):

- Proved that $\rho(t)$ can have at most two local extrema

  $\rightarrow$ Shape of trajectory: greatly restricted ;

  In particular: a single rapid turn (single peak of $|\Omega|$)

- Showed presence of brief tachyonic instability

  - effective entropic mass: $M^2_{(\delta\phi)_\perp} = N^I N^J V_{;IJ} - \Omega^2$
  - tachyonic instability: $M^2_{(\delta\phi)_\perp \equiv s} < 0 \;\; \longleftrightarrow \;\; \eta_\perp^2 \gg 1$

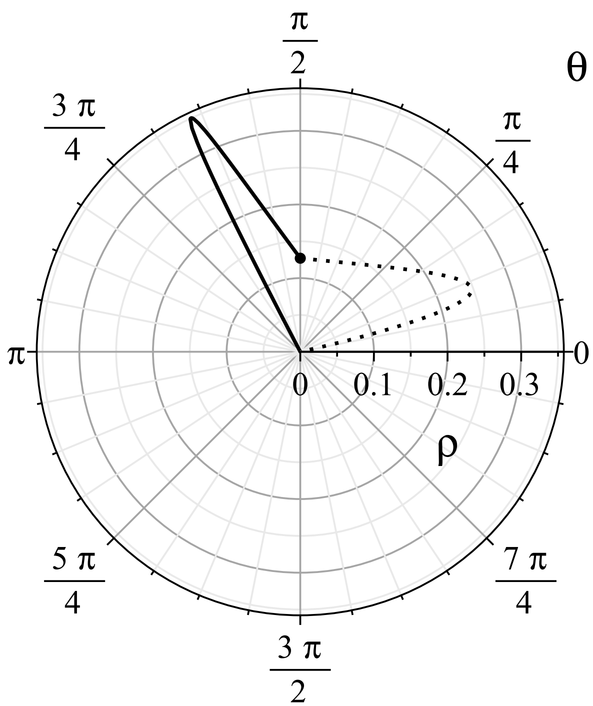
$\rightarrow$ Period δt with $M_s^2 < 0 \;\; \Rightarrow$ desired enhancement of $\mathcal{P}_\zeta$!

Examples of exact solutions:

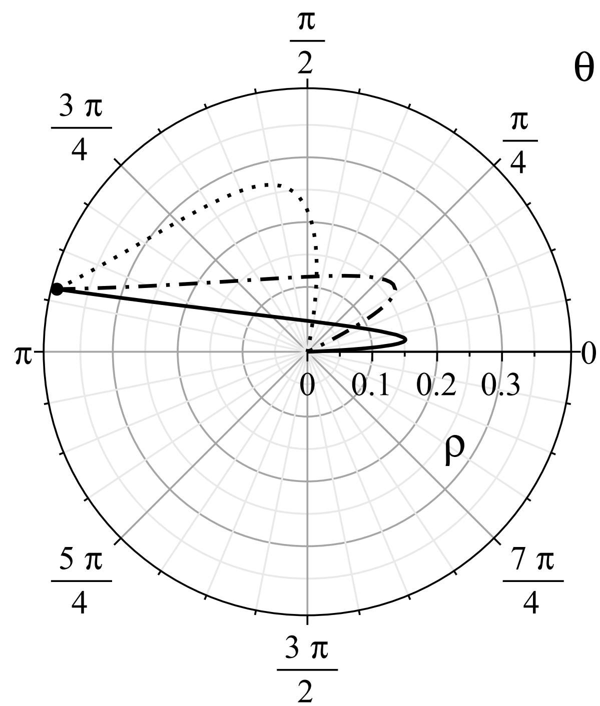
Illustration of all possible types of trajectories on Poincaré disk

[recall: radial variable $\rho \in [0, 1)$]

New result: $\rho(t)$ can have 0 , 1 or 2 local extrema



1 local extremum

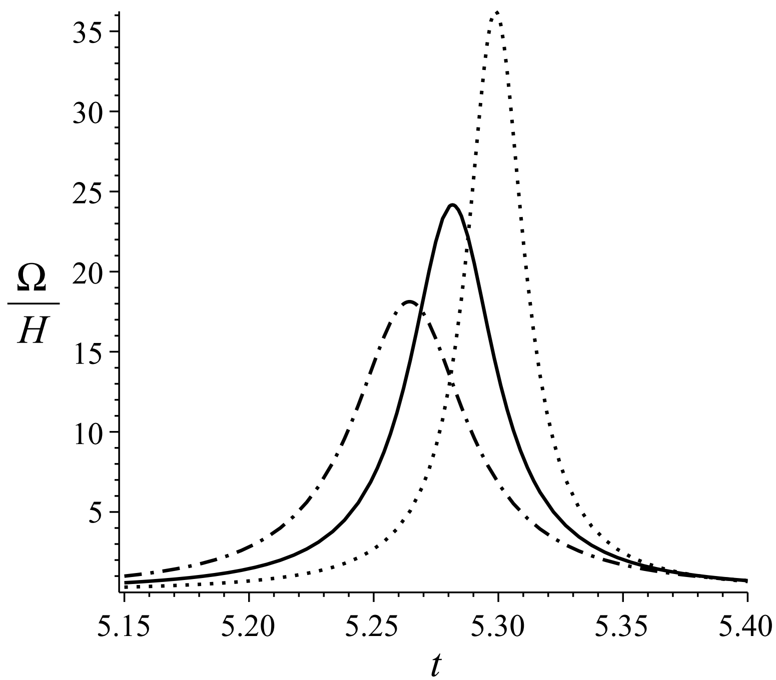


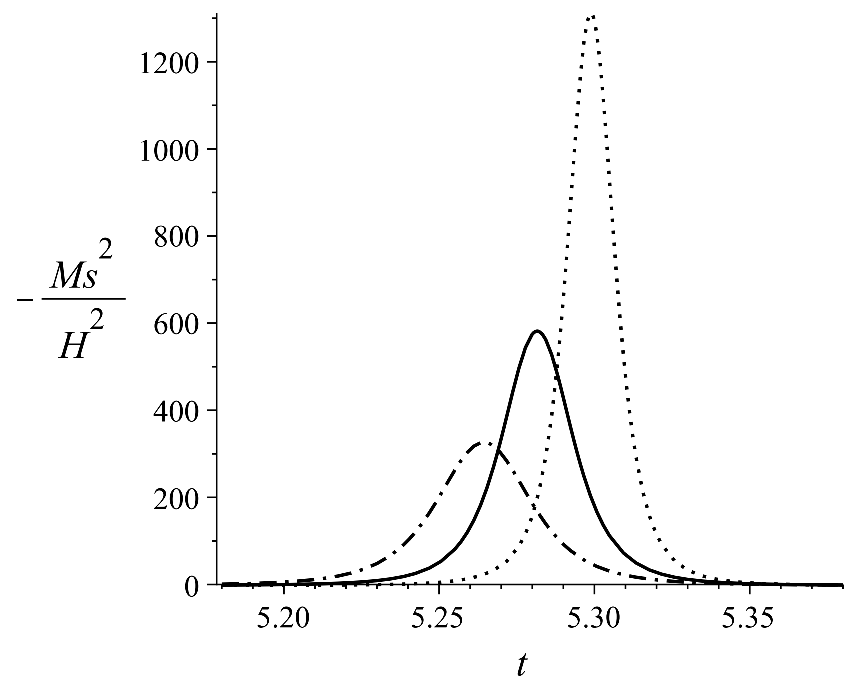
0 or 2 local extrema

Examples of exact solutions:

Illustration of behavior of $\eta_\perp(t) = \frac{\Omega}{H}$ and entropic mass $M_s^2(t)$

New result: transient tachyonic instability





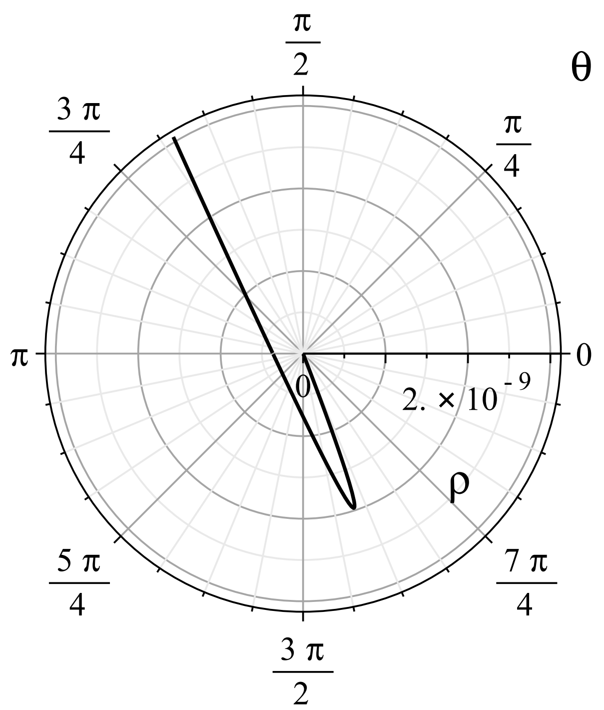
Three examples with number of e-folds $N = \int H dt$ at peak: $\sim$ 11

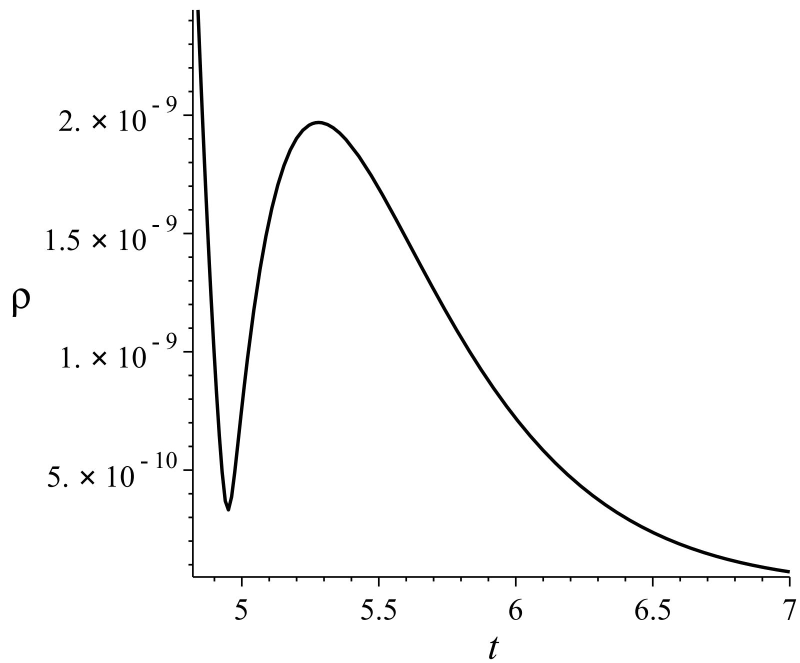
(PBH generation : $|\eta_\perp|_{peak} \sim 25$)

Examples of exact solutions:

Illustration of a typical slow-roll trajectory $(\rho(t), \theta(t))$

New result: $\varepsilon \ll 1$ (slow roll) occurs for $\rho \ll 1$ (equiv. $\varphi \ll 1$)





For comparison: In standard α-attractors slow roll occurs near boundary ($\varphi \to \infty$) of Poincaré disk $\to$ super-super-Planckian excursions in field space

Modified solutions:

Obtained so far: small-field inflation and rapid turn

(Great for PBH-generation !)

BUT: Behavior of $\eta_{||}$-param. - problematic phenomenologically

(On solutions of EoMs: $\eta_{||} = -\ddot{H}/(2H\dot{H})$ - Hubble η-param.)

$\rightarrow$ Need to modify hidden-symmetry solutions

New result: Modified solutions with additional parameter ;

for certain param. value: recover hidden symmetry ;

in general: do not respect the symmetry .

Mod sol.: – Preserve tachyonic inst. and small-field infl.

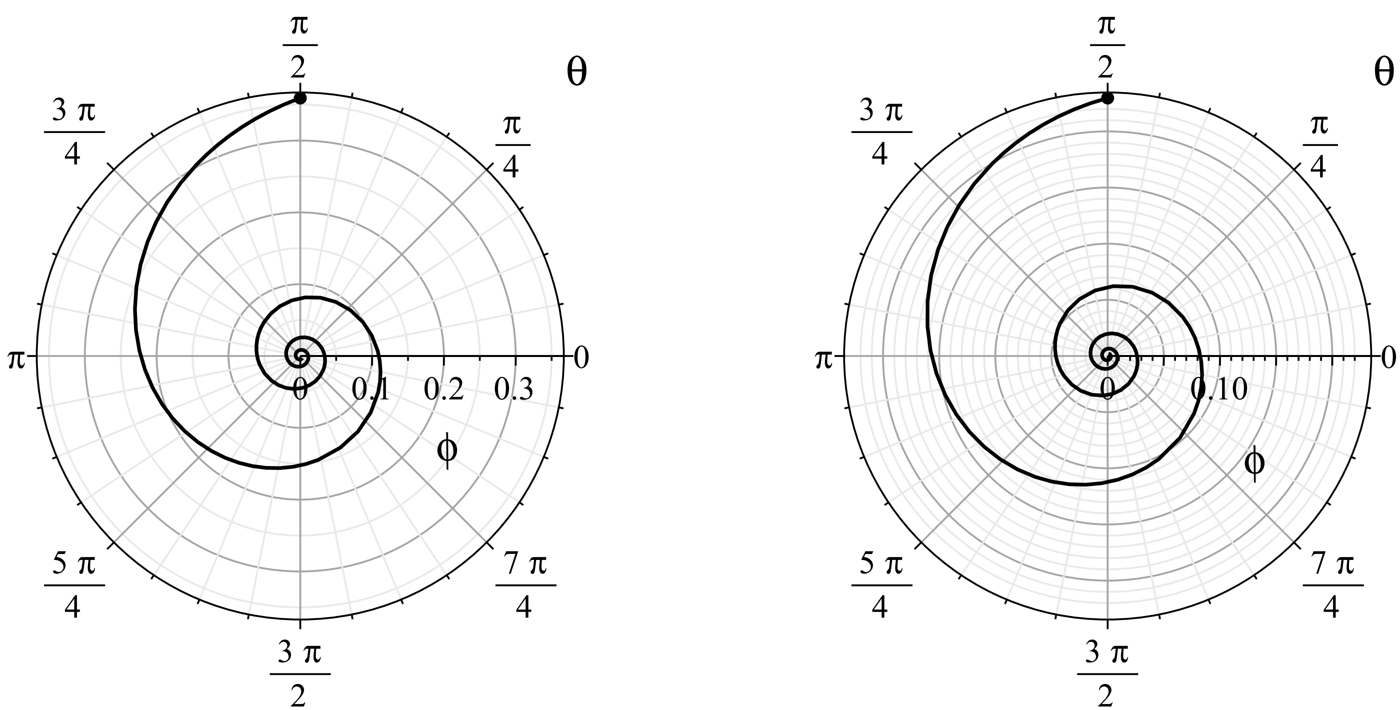
– Phase transition: ultra-slow-roll $\rightarrow$ slow-roll

Dark Energy: exact solutions

Four-param. family of solutions with $\dot{\theta} = const$:

(L.A., J.Dumancic, R.Gass, L.C.R.Wijewardhana, arXiv:2111.12136 [hep-th])

Obtained by taking Poincaré disk and $V = V_{hid.sym.} + const$



Two examples of trajectories $(\varphi(t), \theta(t))$ of the exact solutions

Summary

Found:

- Class of exact two-field inflationary solutions with hidden symmetry leads to PBH generation [trajectory: brief sharp turn]
- Modified solutions with improved Hubble η-parameter
  [Transition between ultra-slow-roll and slow-roll phases]
- Exact solutions describing dark energy [turn rate: always large]

Open questions:

- More general hidden symmetries $\rightarrow$ PBH-generation ?...
- Transitions between other pairs of inflationary phases ?...
- Rapid turns in SUGRA, string compactifications ?...

Thank you!